高考终于结束了，送给大家一道线性代数题暖暖场

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Abstract. 随着 2026 年高考生物结束铃的打响，我结束了三年的高中生活。终于高考结束了，这个暑假有时间回来研究图形学了。大的文章还在施工，作为回归礼，先送给各位一道线性代数题。本题的背景是下篇文章要解析的论文。

Keywords: 线性代数, Math, Computer Graphics, 数学题

1 Problem

Let $S^2 = \{\omega \in \mathbb{R}^3 : \|\omega\| = 1\}$ be the unit sphere equipped with its standard solid angle measure $\mathrm{d}\omega$. Let $D_o : S^2 \to \mathbb{R}$ be a density function on the sphere, and let $M \in \mathbb{R}^{3\times 3}$ be an invertible matrix.

Define a mapping $T : S^2 \to S^2$ by

$$T(\omega) = \omega_o = \frac{M^{-1}\omega}{||M^{-1}\omega||} \tag{1}$$

Suppose a transformed spherical density $D : S^2 \to \mathbb{R}$ is defined by preserving density with respect to solid angle, namely

$$D(\omega)\mathrm{d}\omega = D_o(\omega_o)\,\mathrm{d}\omega \tag{2}$$

Prove that

$$D(\omega) = D_o\left(\frac{M^{-1}\omega}{||M^{-1}\omega||}\right)\frac{\left|\det\left(M^{-1}\right)\right|}{||M^{-1}\omega||^3} \tag{3}$$

Here $\mathrm{d}\omega$ and $\mathrm{d}\omega_o$ denotes infinitesimal solid angle elements on S^2.

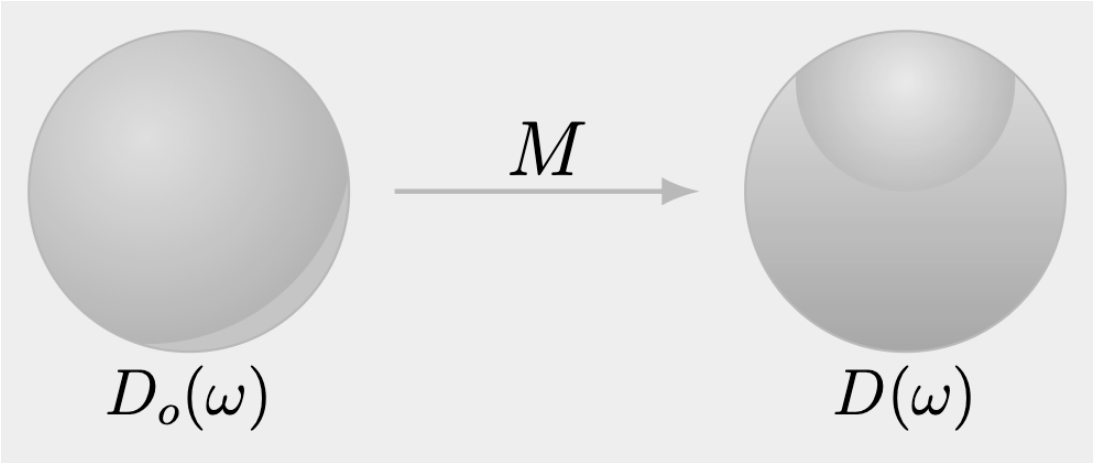

图 1. The transform of the matrix M